HIGH SCHOOL AND UNIVERSITY
MATRICULATION EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1938

ALGEBRA 3

Time—$2\frac{1}{2}$ hours.

Note—Candidates will attempt *all of Section A* and *three questions* of *Section B.*

SECTION A

Values

2 **1.** (*a*) Show that $4\sqrt{3} + 8$, $2 + 2\sqrt{3}$, and 2 are in continued proportion.

3 (*b*) Find the next term of the continued proportion in (*a*).

5 (*c*) A city receives its water supply through a water-main from a reservoir. By what per cent would the diameter of the main have to be increased to double the supply?

2 **2.** (*a*) Prove that if $y = f(x) = \dfrac{ax + b}{cx - a}$, then $x = f(y)$.

2 (*b*) Sketch the graph of $y = \dfrac{x + 2}{x - 1}$.

2 (*c*) Discuss the statement that $\dfrac{x^2 - 1}{x - 1}$ is always equal to $x + 1$.

2 (*d*) If possible, find a value of x for which $\dfrac{x^2 - 1}{x - 1} = 2$.

1 (*e*) Find $\underset{x \to 1}{Lt} \dfrac{x^2 - 1}{x - 1}$.

1 (*f*) Draw the graph of $y = \dfrac{x^2 - 1}{x - 1}$.

5 **3.** (*a*) Given $y = x^3$, find Δy when $x = 1$ and $\Delta x = -0.2$.

5 (*b*) Find $\dfrac{d}{dx}\left(3x^4 - \dfrac{1}{2x} + 7\sqrt{x} - \dfrac{6}{\sqrt{x}}\right)$.

Values

10 **4.** A rectangular tank with open top and square base is to hold 500 cu. ft. Express as a function of the side of the base the cost of lining the tank with lead at 20¢ per square foot. Find the dimensions for which the cost is a minimum, using in the proof the second derivative of the cost.

5 **5.** (*a*) The 8th term of a G.P. is xy^{16} and the 13th is $\dfrac{y^{26}}{x^4}$. Find the 2nd term.

5 (*b*) The cost of drilling a well is as follows:

first 10 feet, 27¢ per foot,
second 10 feet, $28\frac{1}{2}$¢ per foot,
and continues to increase by $1\frac{1}{2}$¢ per foot at every ten feet of depth. Find the cost of drilling a 920-foot well.

7 **6.** Find the cash value of an annuity of $800, deferred five years and running for 20 payments, interest at 5% per annum.

3 **7.** Find the coefficient of x^4 in $(1 - 2x)^2\,(1 + x)^6$.

5 **8.** Solve: $\dfrac{dy}{dx} = 2x - x\sqrt{x} + \dfrac{3}{\sqrt{x}}$.

5 **9.** Find n if ${}^nP_5 = 120\,{}^nC_3$.

SECTION B

Attempt three questions only.

10. Given $y = \sqrt{z}$, and $z = 1 - x^2$,

3 (*a*) Prove Δy is approximately equal to $\dfrac{1}{2\sqrt{z}}\,\Delta z$.

2 (*b*) Express Δz in terms of x and Δx.

2 (*c*) Express Δy in terms of x and Δx.

3 (*d*) Find $\dfrac{d}{dx}\sqrt{1 - x^2}$.

10 **11.** Use integration to find the volume of the solid generated by the rotation, about the axis of x, of the area enclosed by the axis of x, the lines $x = -2$ and $x = 2$, and the portion of the curve $y = 6 - x^2$ which lies between these two straight lines.

Values

10 **12.** Auto license plates are issued showing five digits, the first digit not being zero. How many additional plates could be made if in the third place one of the 26 letters is used instead of a digit?

10 **13.** Given the table of values

x	6	8	10	12	14
y	29	36	45	56	69

use a graph to find a relation of the form $\log y = a + bx$ which will best fit these data.

From this relation compute y when $x = 11$.

10 **14.** Find if possible (*a*) minimum values and (*b*) maximum values, of $\dfrac{x^2 - 3x + 4}{x}$, using in your explanation of each a rough graph of the function.

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